Solving Problems in Physics

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Contents

Introduction

The problems and examinations in this physics course exercise not only your knowledge of physics but also your skill in solving problems. Professional physicists earn their salaries not particularly for their knowledge of physics but for their ability to solve workplace problems. This document presents tips for honing your problem solving skills. These tips and techniques will prove useful to you in your physics courses, in your other college courses, in your career, and in your everyday life.

To set the stage, I want to discuss an example of problem solving from everyday life, namely building a jigsaw puzzle. There are a number of different approaches to building a jigsaw puzzle: My approach is to first turn all the pieces face up, then put together the edge pieces to make a frame, then sort the remaining pieces into piles corresponding to small "sub-puzzles" (blue pieces over here, red pieces over there). I build the sub-puzzles, then piece the sub-puzzles together to build the whole thing. Other people have different approaches to building jigsaw puzzles, but nobody, *nobody*, builds a puzzle by picking up the first piece and putting it in exactly the correct position, then picking up the second piece and putting it in exactly the correct position, and so forth. Solving a jigsaw puzzle involves an approach--a strategy--and a lot of "creative fumbling" as well.

Your physics textbook contains many solved "sample problems". The solutions presented there are analogous to the completed jigsaw puzzle, with every piece in its proper position. No one solves a physics problem by simply writing down the correct equations and the correct reasoning with the correct connections the first time through, just as no one builds a jigsaw puzzle by putting every piece in its correct position the first time through. The "solved problems" in your book are extraordinarily valuable and they deserve your careful study, but they represent the end product of a problem solving session and they rarely show the process involved in reaching that end product. This document aims to expose you to the process.

Solving a physics problem usually breaks down into three stages:

1. Design a strategy.
2. Execute that strategy.
3. Check the resulting answer.

This document treats each of these three elements in turn, and concludes with a summary.

Strategy Design

Look before you leap. Whenever you face a problem, there is an immediate temptation to rush in, roll up your sleeves, and begin tinkering with it. *Resist that temptation*. If you start your detailed work--the execution stage--immediately, you will likely write down a lot of correct statements that do not lead to an answer. Instead, *think* about the problem on an overview level. What sort of conceptual tools will you need to solve the problem? What path will you take to the solution, and in what direction should you start off? Concretely, it often helps to *classify your problem by its method of solution*.

If you are looking for a child lost in the woods, your first step is to sit down, think about what the child probably did and where he probably is, and devise a strategy that will allow you to effectively rescue him. If, instead, you just rush about the woods in random directions, you're likely to become lost yourself.

Where are you now, and where do you want to go? Before you can design a path that takes you from the statement of the problem to its answer, you must be clear about what the situation is and what the goals are. It often helps to *check off* each given datum of the problem, and to *underline* the objective. But for getting an overall sense of the problem, nothing beats summarizing the whole situation with a diagram. The diagram will organize your work and suggest ways to proceed. One of my course graders told me that "When students draw a diagram and label it carefully, they are forced to think about what's going on, and they usually do well. If they just try a globule of math, they mess up."

Keep the goal in sight. Don't get caught in blind alleys that lead nowhere, or even in broad boulevards that lead somewhere but not to where you want to go. It sometimes helps to map a strategy backwards, by saying: "I want to find the answer Z. If I knew Y I could find Z. If I knew X I could find Y . . . " and so forth until you get back to something you are given in the problem statement.

Some students find it useful to make a list of the information given and the goal to be uncovered (e.g. "given the constant acceleration, the initial velocity, and the time, find the displacement"). Others find it sufficient to write down only the goal (e.g. "to find: displacement").

Ineffective strategy. Do not page through your book looking for a magic formula that will give you the answer. Physics teachers do not assign problems in order to torture innocent young minds . . . they assign problems in order to force you into active, intimate involvement with the concepts and tools of physics. Rarely is such involvement provided by plugging numbers into a single equation, hence rarely will you be assigned a problem that yields to this attack. In those rare instances when you do face a problem that can be solved by plugging numbers into a formula, the most effective way to find that formula is by thinking about the physical principles involved, not by flipping through the pages in your book.

Make the problem more specific. You're asked to find the number of ways that M balls can be placed into N buckets. Suppose you can't even begin to map out a strategy. Then try the problem of 3 balls in 5 buckets. Solving the more specific problem will give you clues on how to solve the more general problem. And once you use those clues to solve the more general problem, you can check your solution by trying it out for the already-solved special case M=3 and N=5.

Large problems. At times you will be faced with big problems for which no method of solution is immediately apparent. In this case, break your problem into several smaller subproblems, each of which is simple enough that you know how to solve it. At this strategy-design stage it is not important that you actually solve the subproblems, but rather that you know you can solve them. You might begin by mapping out a strategy that leads nowhere, but then you haven't wasted time by implementing this strategy. Once you have mapped out a strategy that leads from the given information to the answer, you can then go back and execute the calculations. This strategy has been known from the time of the ancients under the name of "divide and conquer".

Execution (Tactics)

Eventually, of course, you *do* have to roll up your sleeves and tinker with the problem. As you do so, keep your strategy in mind, and keep the following tips in mind as well:

Work with symbols. Depending on the problem statement, the final answer might be a formula or a number. In either case, however, it's usually easier to work the problem with symbols and plug in numbers, if requested, only at the very end. There are three reasons for this: First, it's easier to perform algebraic manipulations on a symbol like "m" than on a value like "2.59 kg". Second, it often happens that intermediate quantities cancel out in the final result. Most important, expressing the result as an equation enables you to examine and understand it (see the section on "Answer Checking") in a way that a number alone does not permit.

(Working with symbols instead of numbers can lead to confusion as to which symbols represent given information and which represent unknown desired answers. You can resolve this difficulty by remembering--as recommended above--to "keep the goal in sight".)

Define symbols with mnemonic names. If a problem involves a helium atom colliding with a gold atom, then define m_h as the mass of the helium atom and m_g as the mass of the gold atom. If you instead pick the symbols m_1 and m_2, you stand a good chance of mixing up the symbols and their meanings as you solve the problem. And if you don't define the symbols at all, but just begin throwing around m's and M's, you'll confuse both yourself and whoever is grading your answer.

Keep packets of related variables together. In acceleration problems, the quantity $(1/2)at^2$ comes up over and over again. This collection of variables has a simple physical interpretation, transparent dimensions, and a convenient memorable form. In short, it is easy to work with as a packet. Take advantage of this ease. Don't artificially divide this packet into pieces, or write it in an unfamiliar form like $t^2a/2$. Packets like this come up in all aspects of physics--some are even given names (e.g. "the Bohr radius" in atomic physics). Look for these packets, think about what they are telling you, and respect their integrity.

Neatness and organization. I am not your mother, and I will not tell you how to organize either your dorm room or your problem solutions. But I can tell you that it is easier to work from neat, well-organized pages than from scribbles. I can also warn you about certain handwriting pitfalls: Distinguish carefully between t and +, between l and 1, and between Z and 2. (I write a t with a hook at the bottom, an l in script lettering, and a Z with a cross bar. You can form your own conventions.) These suggestions on neatness, organization, and handwriting do not arise from prudishness--they are practical suggestions that help avoid algebraic errors, and they are for your benefit, not mine. (On the other hand, it doesn't hurt to be neat and organized for the benefit of your grader. One course grader of mine pointed out: "If I can't read it, I can't give you credit.")

Avoid needless conversions. If the problem gives you one length in meters and another in inches, then it's probably best to convert all lengths to meters. But if all the lengths are in inches, then there's no need to convert everything to meters--your answer should be in inches. In fact, you might not actually need to convert. For example, perhaps two lengths are given in inches and the final answer turns out to depend only on the ratio of those two lengths. In that case, the ratio is the same whether the lengths going into the ratio are inches or meters. It's easy to make arithmetic errors while doing conversions. If you don't convert, then you don't make those errors!

Keep it simple. I will not assign baroque problems that require tortuous explanations and pages of algebra. If you find yourself working in such a way, then you're on the wrong path. The cure is to stop, go back to the beginning, and start over with a new strategy. (Generations of students have kept track of this rule by remembering to KISS: Keep It Simple and Straightforward.)

Answer Checking

Checking your answer does not mean comparing it to the answer in the back of the book. It means finding the characteristics of your answer and comparing them to the characteristics that you expect. Some of your problems--particularly the ones assigned early in the course--will actually lead you through the checking stage in order to familiarize you with the process. Other problems will leave it to you to perform this check. In either case, checking your answer is not just good problem solving practice that helps you gain points on problem assignments and on exams. The checking stage builds familiarity with the content of physics and the character of problem solutions, and hence develops your intuition to make solving other problems--and learning more physics--easier. (See Daniel F. Styer, "Guest comment: Getting there is half the fun", *American Journal of Physics* **64** (1998) 105-106.)

Dimensional analysis. Suppose you find a formula for distance (in, say, meters) in terms of some information about velocity (meters/second), acceleration (meters/second^2), and time (seconds). If your formula is correct then all of the dimensions on the right hand side must cancel so as to end up with "meters".

Numerical reasonableness. If your problem asks you to find the mass of a squirrel, do you find a mass of 1,970 kilograms? Even worse, do you find a mass of -1,970 kilograms?

[**Reasonable speeds.** "My calculations give me a speed of 23 m/s. Is this reasonable?" It's hard for most people to get a feel for the reasonableness of speeds expressed in meters per second. Until this qualitative feel develops, Americans should check for reasonableness by converting speeds in meters per second to speeds in miles per hour: simply double the number (20 m/s is about 40 mi/hr). Non-Americans should convert to kilometers per hour: simply quadruple the number (20 m/s is about 80 km/hr).]

Algebraically possible. Would evaluating your formula ever lead you to divide by zero or take the square root of negative number?

Functionally reasonable. Does your answer depend on the given quantities in a reasonable way? For example, you might be asked how far a projectile travels after it is launched at a given speed with a given angle. Common sense says that if the initial speed is increased (keeping the angle constant) then the distance traveled will increase. Does your formula agree with common sense?

Limiting values and special cases. In the projectile travel distance problem mentioned above, the range is obviously zero for a vertical launch. Does your formula give this result? If you solve a problem regarding two objects, does it give the proper result when the two objects have equal masses? When one of them has zero mass (i.e. does not exist)?

Symmetry. Problems often have geometrical symmetry from which you can determine the direction of a vector but not its magnitude. More often they have a "permutation" symmetry: If your problem has two objects, you can call the cube "object number 1" and the sphere "object number 2" but your final answer had better not depend upon how you numbered your objects. (That is, it should give the same answer if every "1" is changed to a "2" and vice versa.)

Specify units. "The distance is 5.72" is not an answer. Is that 5.72 miles, 5.72 meters, or 5.72 inches? Similarly, if the answer is a vector, both magnitude and direction must be specified. (The direction may be drawn into a diagram rather than stated explicitly.)

Significant figures. Any number that comes from an experiment comes with some uncertainty. Most of the numbers in this course come with three significant figures. If a ball rolls 3.24 meters in 2.41 seconds, then report its speed as 1.34 m/s, not 1.34439834 m/s. Most introductory physics courses do not require a formal or technical error analysis, but you should avoid inaccurate statements like the second quotient above.

Large problems. If you break up your large problem into several subproblems, as recommended above, then check your results at the end of each subproblem. If your answer to the second subproblem passes its checks, but your answer to the third subproblem fails its checks, then your execution error almost certainly falls within the third subproblem. Knowing its general location, you can quickly go back and correct the error, so its effects will not propagate on to the remaining subproblems. This can be a real time-saver.

Summary

The problems in your physics course can be fun and exciting. Approach them in the spirit of exploration and they will not disappoint you!

1. Strategy design
    a. Classify the problem by its method of solution.
    b. Summarize the situation with a diagram.
    c. Keep the goal in sight (perhaps by writing it down).
2. Execution tactics
    a. Work with symbols.
    b. Keep packets of related variables together.
    c. Be neat and organized.
    d. Keep it simple.
3. Answer checking
    a. Dimensionally consistent?
    b. Numerically reasonable (including sign)?
    c. Algebraically possible? (Example: no imaginary or infinite answers.)
    d. Functionally reasonable? (Example: greater range with greater initial speed.)
    e. Check special cases and symmetry.
    f. Report numbers with units specified and with reasonable significant figures.

Further Reading

The classic exploration of mathematical problem solving technique is

- George Polya, *How To Solve It* (Princeton University Press, Princeton, New Jersey, 1957).

More mundane and somewhat pedantic, but nevertheless valuable, is

- Donald Scarl, *How To Solve Problems: For Success in Freshman Physics, Engineering, and Beyond*, third edition (Dosoris Press, Glen Cove, New York, 1993).

Study of the following books will help develop your general (as opposed to strictly mathematical) problem-solving skills:

- James L. Adams, *Conceptual Blockbusting: A Guide to Better Ideas* (Norton, New York, 1980),
- Berton Roueche, *The Medical Detectives* (Times Books, New York, 1980) and *The Medical Detectives, volume II* (Dutton, New York, 1984),
- Martin Gardner, *Aha! Insight* (Freeman, New York, 1978),
- Donald J. Sobol, *Two-Minute Mysteries*,
- Arthur Conan Doyle, Sherlock Holmes stories,
- Agatha Christie, Hercule Poirot stories, particularly *Murder on the Orient Express*.

Entry into recent literature on physics problem solving skills is provided by

- Frederick Reif, "Understanding and teaching important scientific thought processes", *American Journal of Physics* **63** (1995) 17-35 (especially section V),
- Rolf Plotzner, *The Integrative Use of Qualitative and Quantitative Knowledge in Physics Problem Solving* (Peter Lang, Frankfurt am Main, 1994).