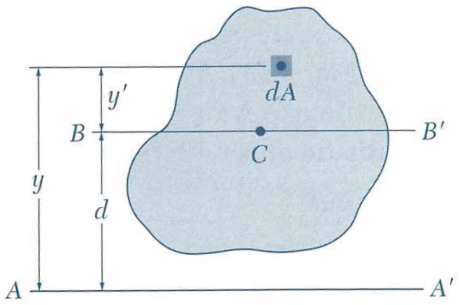
Parallel Axis Theorem

$$I = \bar{I} + Ad^2$$

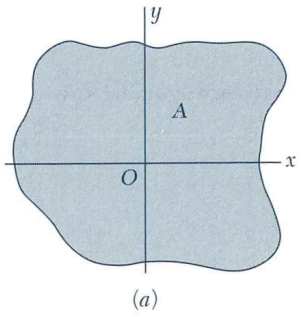
The moment of inertia of a shape about an arbitrary axis is equal to the moment of inertia of the shape about a parallel axis passing through the centroid of the shape, plus a correction factor Ad^2 where A is the area of the shape, and d is the distance between the two parallel axes.



Radius of Gyration

$$k = \sqrt{\frac{I}{A}}$$

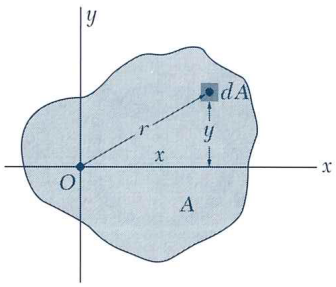
The radius of gyration, k, is distance from an axis at which, if the entire area was concentrated there, would produce the same moment of inertia about that axis as the the original shape does.



Polar Moment of Inertia

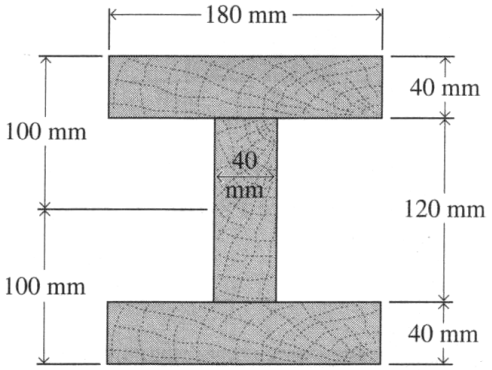
The polar moment of inertial, J_O is a measure of the ability of a shaft to resist torsion in the same way that rectangular moment of inertia is related to bending.

$$J_O = \int r^2 dA = I_x + I_y$$



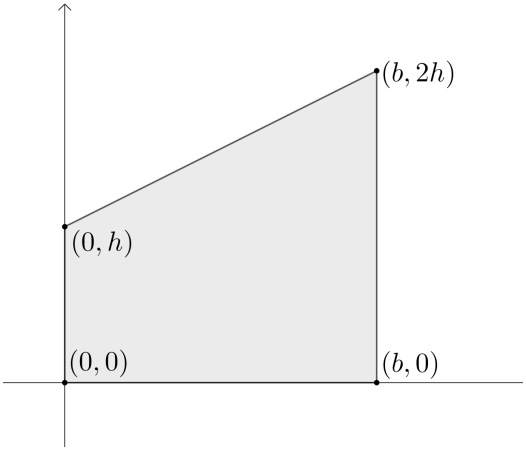
Example 1

Determine the moment of inertia and radius of gyration about a horizontal axis passing through the base.



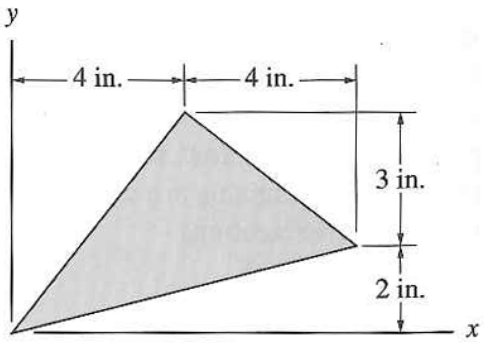
Example 2

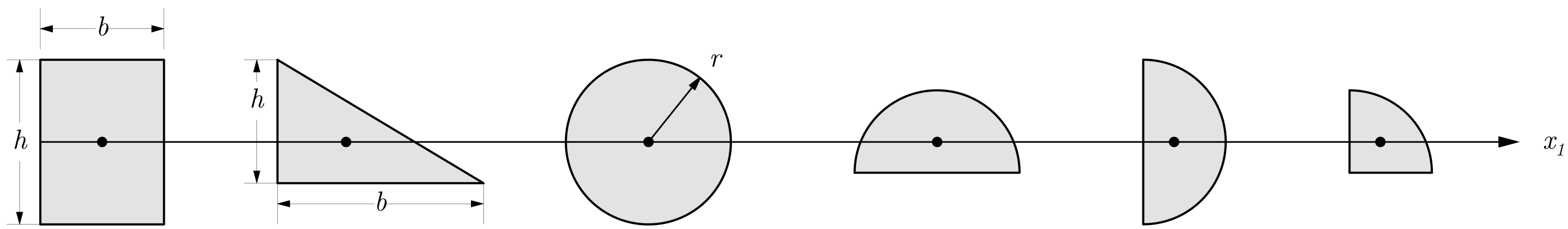
Determine the moment of inertia about the *x*- and *y*- axes. Express your answer in terms of *b* and *h*, and simplify completely



Example 3

Determine the moment of inertia and radius of gyration about the *x*-axis.





$\bar{I}_{x_1} = \frac{1}{12}bh^3$

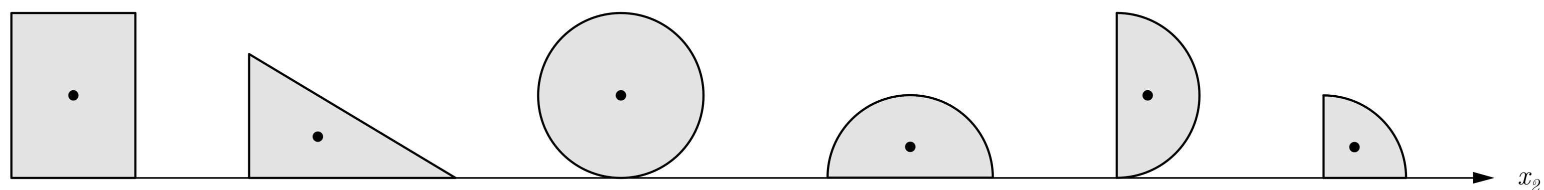
$\bar{I}_{x_1} = \frac{1}{36}bh^3$

$\bar{I}_{x_1} = \frac{\pi r^4}{4}$

$\bar{I}_{x_1} = \frac{\pi r^4}{8} - \frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2 = 0.1098r^4$

$\bar{I}_{x_1} = \frac{\pi r^4}{8}$

$\bar{I}_{x_1} = \frac{\pi r^4}{16} - \frac{\pi r^2}{4}\left(\frac{4r}{3\pi}\right)^2 = 0.0548r^4$



$I_{x_2} = \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2 = \frac{bh^3}{3}$

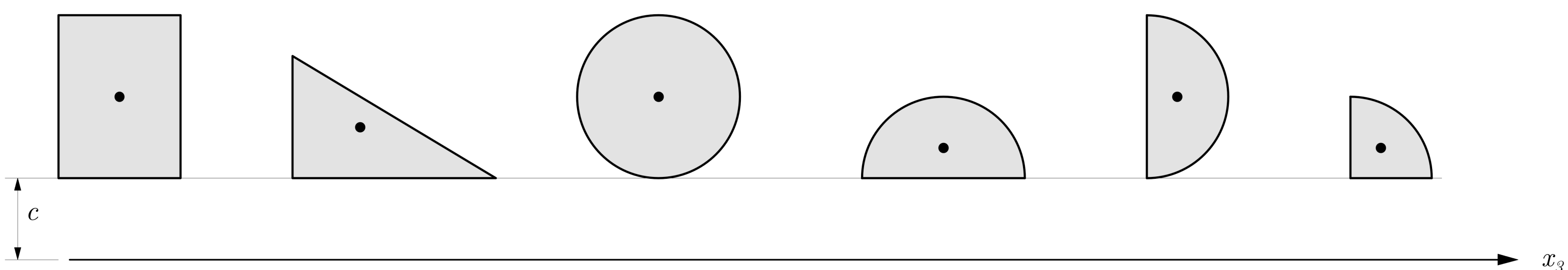
$I_{x_2} = \frac{bh^3}{36} + \frac{bh}{2}\left(\frac{h}{3}\right)^2 = \frac{bh^3}{12}$

$I_{x_2} = \frac{\pi r^4}{4} + \pi r^2 (r)^2 = \frac{5\pi r^4}{4}$

$I_{x_2} = \frac{\pi r^4}{8}$

$I_{x_2} = \frac{\pi r^4}{8} + \frac{\pi r^2}{2}(r)^2 = \frac{5\pi r^4}{8}$

$I_{x_2} = \frac{\pi r^4}{16}$



$I_{x_3} = \frac{bh^3}{12} + bh\left(\frac{h}{2} + c\right)^2$

$I_{x_3} = \frac{bh^3}{36} + \frac{bh}{2}\left(\frac{h}{3} + c\right)^2$

$I_{x_3} = \frac{\pi r^4}{4} + \pi r^2 (r + c)^2$

$I_{x_2} = 0.1098r^4 + \frac{\pi r^2}{2}\left(\frac{4r}{3\pi} + c\right)^2$

$I_{x_3} = \frac{\pi r^4}{8} + \frac{\pi r^2}{2}(r + c)^2$

$I_{x_3} = 0.0549r^4 + \frac{\pi r^2}{4}\left(\frac{4r}{3\pi} + c\right)^2$