Important equations:

Moment of Inertia with respect to the x-axis: $I_x = \int y^2 dA$

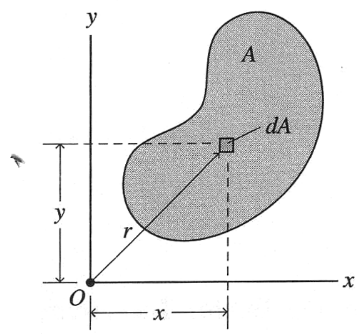
Moment of Inertia with respect to the y-axis: $I_y = \int x^2 dA$

Polar Moment of Inertia: $J_o = \int r^2 dA$

Radius of Gyration with respect to the x-axis: $k_x = \sqrt{\frac{I_x}{A}}$

Radius of Gyration with respect to the y-axis: $k_y = \sqrt{\frac{I_y}{A}}$

Parallel Axis Theorem: $I = \bar{I} + Ad^2$



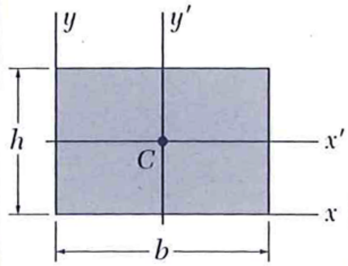
Notes:

1. In mechanics we are concerned with the *area* moment of inertia; don't confuse it with the *mass* moment of inertia used in rotational dynamics. The moment of inertia is also called the *second moment of area*.
2. Moments of inertia are a property of a shape and an axis. They are a measure of how the area is distributed. Areas located farther away from the axis are more important by a factor of the square of the distance away.
3. Moments of inertia occur in the analysis of situations where a force varies linearly with distance from an axis, for example: forces on a submerged surface, or when calculating bending stresses in a beam.
4. Moments of inertia have units of *Length to the* 4th *power*, and are always positive.
5. An over bar indicates a *centroidal* moment of inertia, referring to a moment of inertia about an axis passing through the area's centroid.
6. The centroidal moment of inertia is the is the smallest moment of inertia for any particular axis orientation. Moments of inertia get larger as the the axis moves away from the centroid.
7. When using integration to find Moment of Inertia, use strips which are parallel to the axis you are calculating about; e.g., use horizontal strips to find I_x.
8. The formulas for moment of inertia include a length times a length cubed (bh^3). The dimension which is cubed is always the distance perpendicular to the axis.
9. The *radius of gyration* gives the RMS (root mean square) average distance to the parts of the shape.
10. The radius of gyration has units of Length.
11. The parallel axis theorem can be used to determine moments of inertia of a shape about any axis if you know the moment of inertia about a parallel axis passing through the shape's centroid. It is very useful. Make sure you remember the formula and what each term means.

Moments of Inertia of Common Geometric Shapes

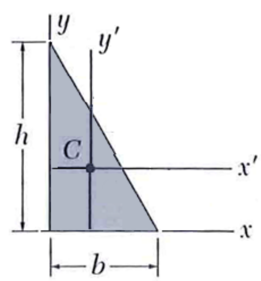
Rectangle

$$\bar{I}_{x'} = \frac{1}{12}bh^3$$
$$\bar{I}_{y'} = \frac{1}{12}b^3h$$
$$I_x = \frac{1}{3}bh^3$$
$$I_y = \frac{1}{3}b^3h$$
$$J_C = \frac{1}{12}bh(b^2 + h^2)$$



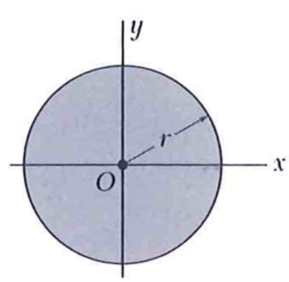
Triangle

$$\bar{I}_{x'} = \frac{1}{36}bh^3$$
$$\bar{I}_{y'} = \frac{1}{36}b^3h$$
$$I_x = \frac{1}{12}bh^3$$
$$I_y = \frac{1}{12}b^3h$$



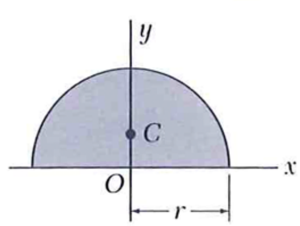
Circle

$$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$$
$$J_O = \frac{1}{2}\pi r^4$$



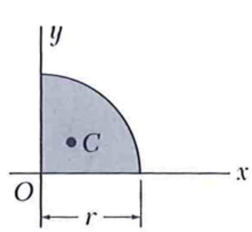
Semicircle

$$I_x = \bar{I}_y = \frac{1}{8}\pi r^4$$
$$J_O = \frac{1}{4}\pi r^4$$

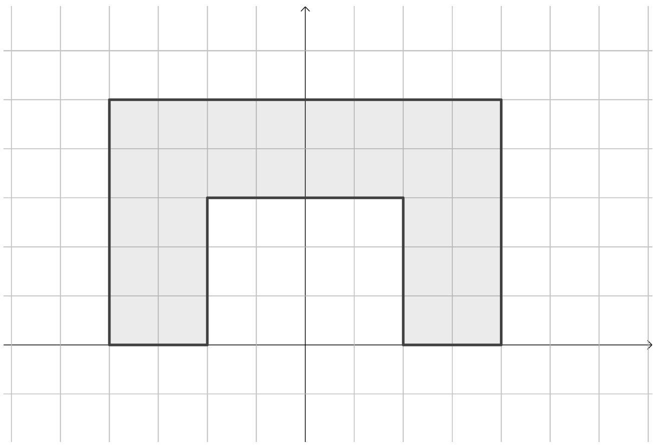


Quarter circle

$$I_x = I_y = \frac{1}{16}\pi r^4$$
$$J_O = \frac{1}{8}\pi r^4$$



Example 1: Determine the moment of inertia of the shape shown about the *x*- and *y*- axes. Grid units are inches.



Example: Determine the moment of inertia of the shape shown about the x- and y- axes. Grid units are inches.

Explain why the results are the same or different than the results found in the previous example.