How to determine centriods by integration

1. Know the formulas:

$$A = \int dA \qquad \bar{x} = \frac{Q_y}{A}$$
$$Q_x = \int \bar{y}_{el}\, dA \qquad \bar{y} = \frac{Q_x}{A}$$
$$Q_y = \int \bar{x}_{el}\, dA$$

2. Draw and label a sketch of the situation.

3. Determine the bounding functions that delimit the area.

4. Select vertical or horizontal strips and draw a representative strip on your sketch. Try to pick a strip that will simplify the math.

5. Determine expressions for dA, $\bar{x}_{el}$, and $\bar{y}_{el}$,which depend on the strips you've chosen.

6. Determine limits of the integrals.

7. For each of the three integral equations:

   a. Substitute your expressions for dA, $\bar{x}_{el}$, and $\bar{y}_{el}$ into the integrals.

   b. Substitute in the bounding function to make the equation integratabtle.

   c. Perform the integral.

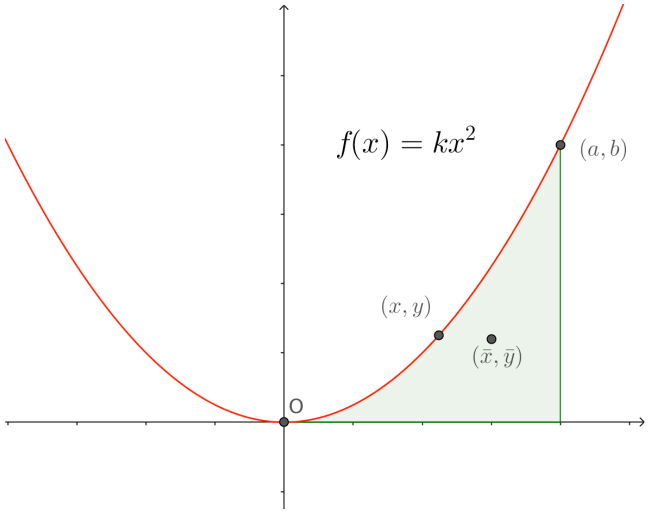
   d. Evaluate the integral between upper and lower limits and simplify.

8. Substitute results into the formulas for $\bar{x}$ and $\bar{y}$ to get the result.

Note: If you are only looking for $\bar{x}$, or $\bar{y}$, you will only need to solve two integrals. Sometimes symmetry will help you simplify the problem.

Example 1

Find the coordinates of the centroid of the parabolic spandrel shown.



Homework

Find the coordinates of the centroid of the general spandrel shown.

This is similar to the previous problem except that exponent n is unspecified and can take any real value; constant k depends on a, b, and n. The centroidal coordinates will be a function of n.