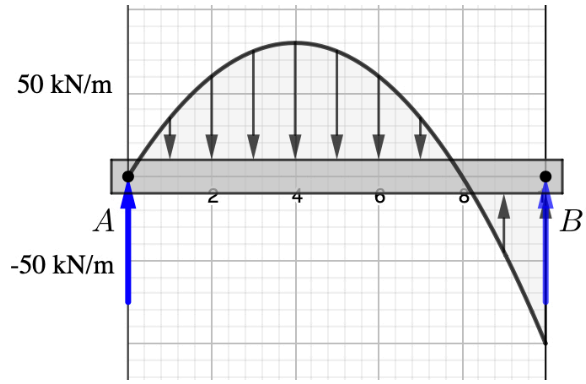
A 10 m long beam is subjected to a parabolic distributed load which varies according to this relationship:

$$w(x) = (-5x^2 + 40x) \text{ kN/m} \qquad L = 10 \text{ m}$$

Draw the shear and bending moment diagrams and determine: the reactions at A and B, and the maximum (signed) values of shear and bending moment.



1. Determine the total downward force due to the distributed load. Consider the load to be made of an infinite number of infinitesimal weights $dW = w(x)\,dx$.

$$\begin{aligned} W &= \int_0^L w(x)\,dx \\ &= \int_0^L (-5x^2 + 40x)\,dx \\ &= -\frac{5}{3}x^3 + 20x^2 \Big|_0^{10} \\ &= 333.3 \text{ kN} \end{aligned}$$

2. Determine the reactions at A and B. Note when taking moments each infinitesimal weight is multiplied by its perpendicular distance from point A.

$$\begin{aligned} \Sigma M_A &= 0 \\ B(L) - \int_0^L x\,w(x)\,dx &= 0 \\ B &= \frac{\int_0^L -5x^3 + 40x^2\,dx}{L} \\ &= \frac{-\frac{5}{4}x^4 + \frac{40}{3}x^3 \Big|_0^{10}}{10} \\ &= 83.3 \text{ kN} \end{aligned}$$

$$\begin{aligned} \Sigma F_y &= 0 \\ A &= W - B \\ &= 250 \text{ kN} \end{aligned}$$

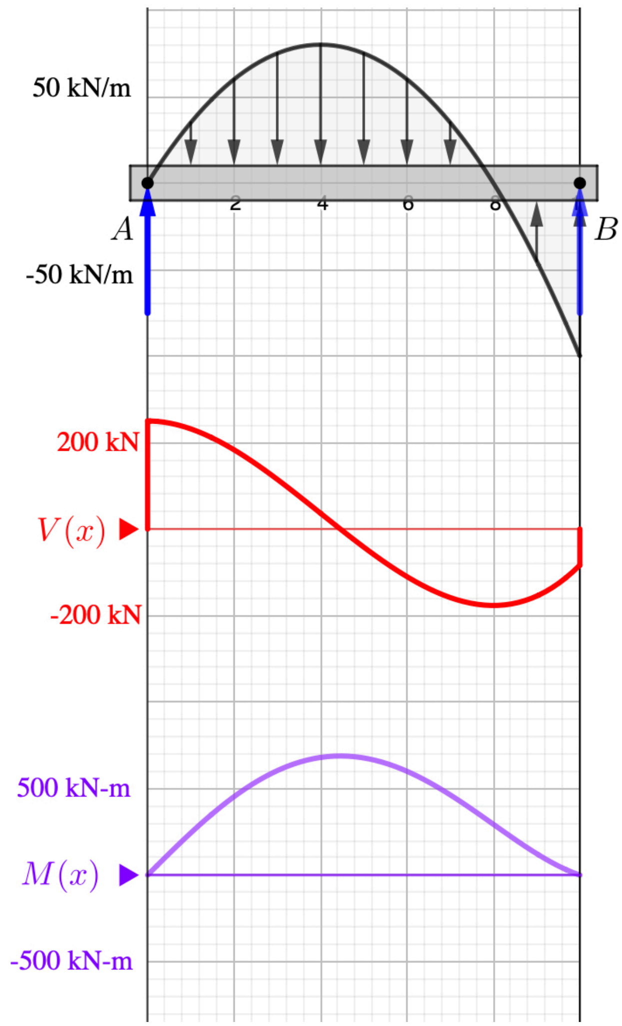
3. Integrate the load equation to get the shear function. Use boundary condition at $x = 0$ to determine the constant of integration. (The shear at $x = 0$ is reaction force A.)

$$\begin{aligned} \frac{dV}{dx} &= -w(x) \\ \int dV &= -\int (5x^2 - 40x)\,dx \\ V(x) &= \frac{5}{3}x^3 - 20x^2 + C_1 \qquad V(0) = A, \text{ so } C_1 = 250 \text{ kN} \\ &= \frac{5}{3}x^3 - 20x^2 + 250 \qquad \text{(kN)} \end{aligned}$$

4. Similarly, integrate the shear function to get the moment function and apply the boundary condition at $x = 0$. (There is no moment at end A.)

$$\frac{dM}{dx} = V(x)$$
$$\int dM = \int \left(\frac{5}{3}x^3 - 20x^2 + 250\right)\, dx$$
$$M(x) = \frac{5}{12}x^4 - \frac{20}{3}x^3 + 250x + C_2 \qquad M(0) = 0,\ \text{so } C_2 = 0$$
$$= \frac{5}{12}x^4 - \frac{20}{3}x^3 + 250x \qquad \text{(kN-m)}$$

5. Plot the equations of shear and bending moment.



6. By inspection, the maximum shear occurs at the left hand end and is equal to the value of reaction A.

$$V_{max} = A = 250 \text{ kN}$$

7. The maximum moment occurs where the derivative of the moment function is zero, so find this point (use technology or trial and error).

$$\frac{dM}{dx} = 0$$
$$V(x) = 0,\ \text{since } \frac{dM}{dx} = V(x)$$
$$\frac{5}{3}x^3 - 20x^2 + 250 = 0$$
$$\text{Roots: } x = \begin{cases} -3.147, \\ 4.460, \\ 10.687 \end{cases}$$

8. Evaluate the moment at $x = 4.46$ m, which is the only root located between the ends of the beam.

$$M_{max} = M(4.46) = 688.4 \text{ kN-m}$$