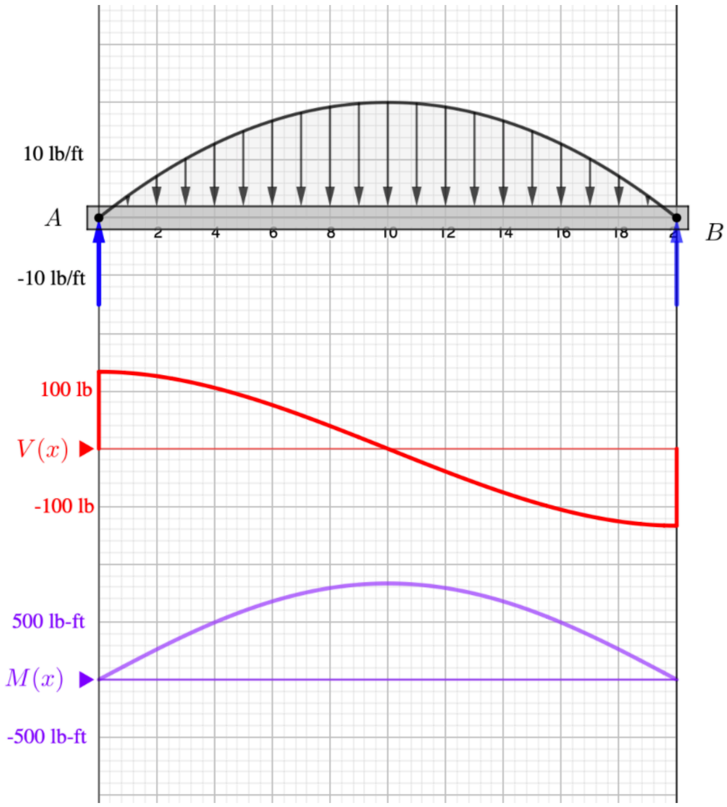
A 20 ft long beam is supporting a distributed load which varies according to the relation

$$w(x) = -\frac{x^2}{5} + 4x \quad \text{(lb/ft)}$$

A. Determine the reactions at A and B.
B. Determine the equations of shear and bending moment as functions of x.
C. Determine the value of the maximum internal moment.



The equivalent downward force of the distributed load is the "area" under the loading curve.

$$\begin{aligned} W &= \int_0^{20} w(x)dx \\ &= \int_0^{20} \left(-\frac{x^2}{5} + 4x\right) dx \\ &= -\frac{x^3}{15} + 2x^2 \Big|_0^{20} \\ &= 266.7 \text{ lb} \end{aligned}$$

Since the load is symmetrical, the reactions are equal and A and B each support half the load.

$$A = B = W/2 = 133.3 \text{ lb}$$

To find the shear function use the relation between shear and load:

$$\frac{dV}{dx} = -w(x)$$
$$dV = -\left(-\frac{x^2}{5} + 4x\right)dx$$

Integrating both sides:

$$\begin{aligned} V(x) &= \int\left(\frac{x^2}{5} - 4x\right)dx \\ &= \frac{x^3}{15} - 2x^2 + C_1 \\ &= \frac{x^3}{15} - 2x^2 + 133.3 \end{aligned}$$

The constant is found because we know that at $V(0) = A = 133.3$ lb.

The moment function is found similarly with $M(0) = 0 \therefore C_2 = 0$

$$\begin{aligned} M(x) &= \int V(x)dx \\ &= \int\left(\frac{x^3}{15} - 2x^2 + 133.3\right)dx \\ &= \frac{x^4}{60} - \frac{2x^3}{3} + 133.3x + C_2 \\ &= \frac{x^4}{60} - \frac{2x^3}{3} + 133.3x \end{aligned}$$

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The maximum moment occurs where V(x) = 0, i.e., the midpoint. Evaluating:

$$\begin{aligned} M(10) &= \frac{10^4}{60} - \frac{(2)10^3}{3} + (133.3)10 \\ &= 833 \text{ ft-lb} \end{aligned}$$